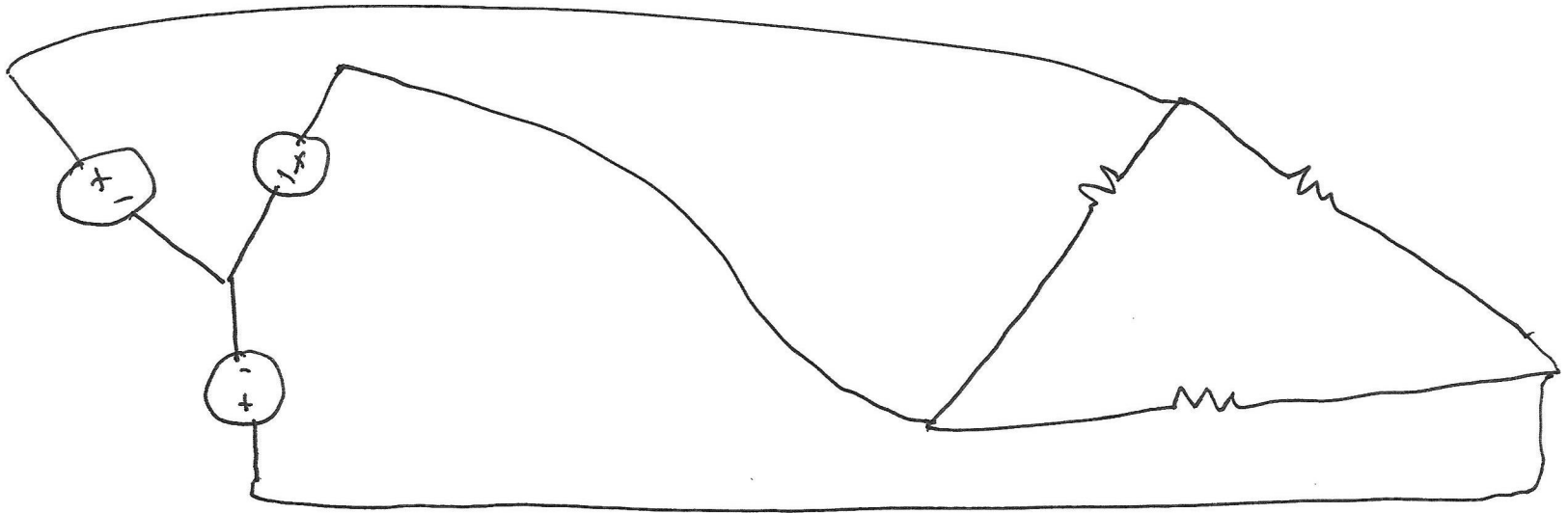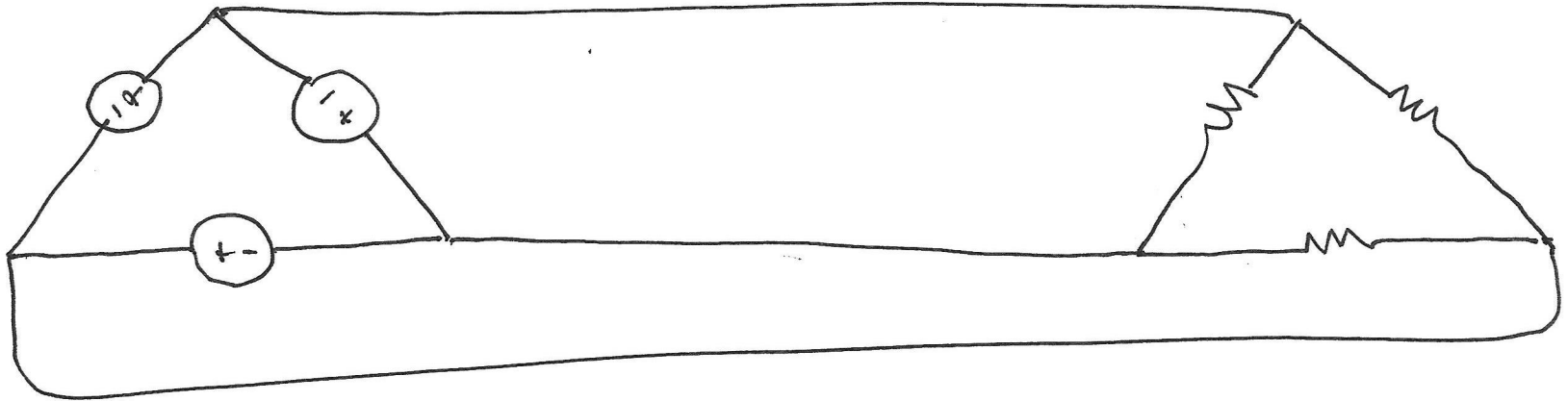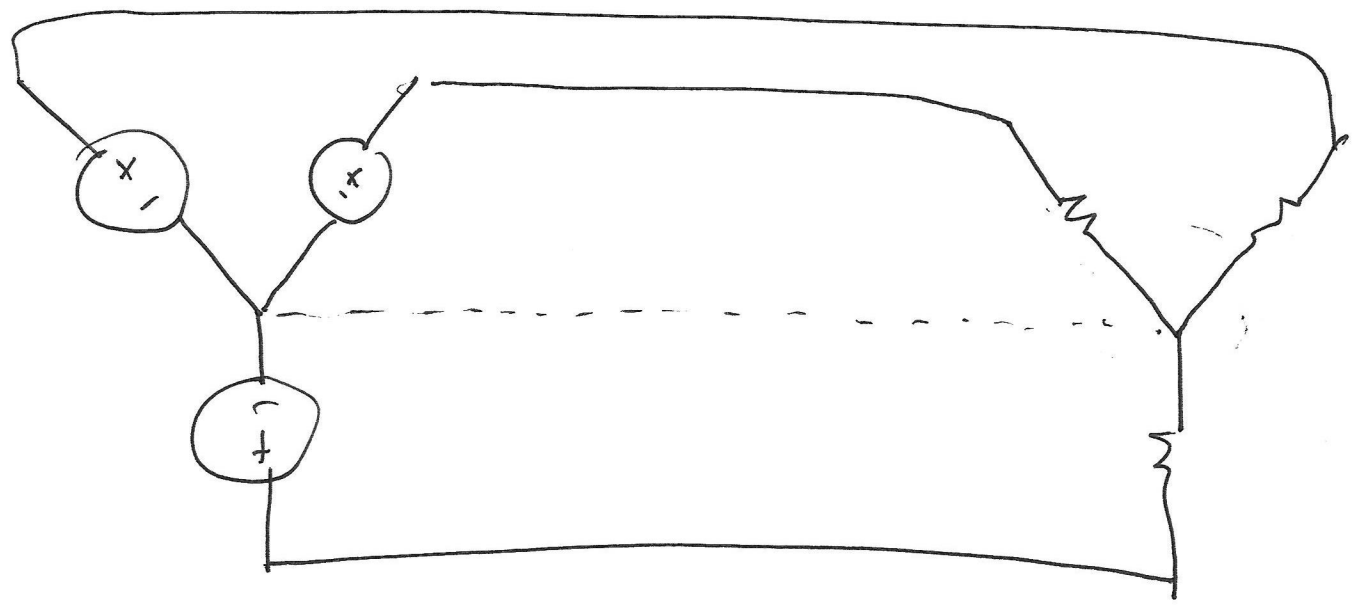Bode $\Rightarrow$ dB vs log f

$dB = 20 \log_{10} |H(f)|$

decibels

3

Delta-to-Wye and Wye-to-Delta Transformations

In the process of circuit analysis we often encounter three-terminal subcircuits such as those shown below. Many times it would simplify the analysis procedure if we could convert the "delta" (also called "pi") connected circuit to an equivalent "wye" (also called "tee") configuration, or conversely, convert the wye to an equivalent delta.
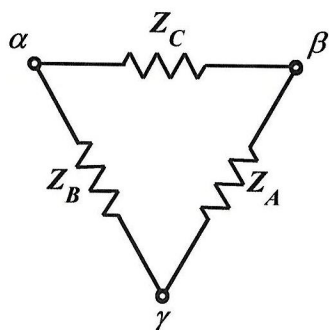
Delta-Connected Impedances
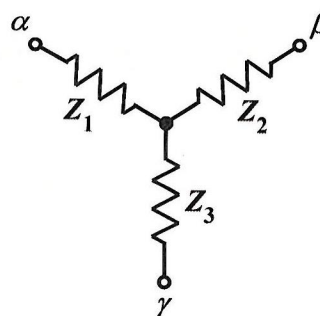
Wye-Connected Impedances

The transformation equations are shown below. The objective of this note is to derive those equations and therefore show that the transformations do indeed result in equivalent subcircuits — that is, we can replace one subcircuit with the other, and it will be impossible to determine any difference in behavior with respect to the terminal connections; they behave identically.

$\Delta \to Y$ Transformation

Converting from a delta configuration to a wye configuration is accomplished as follows:

$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

Y → Δ Transformation

Similarly, a wye-connected subcircuit can be easily converted to a delta-connected configuration as follows:
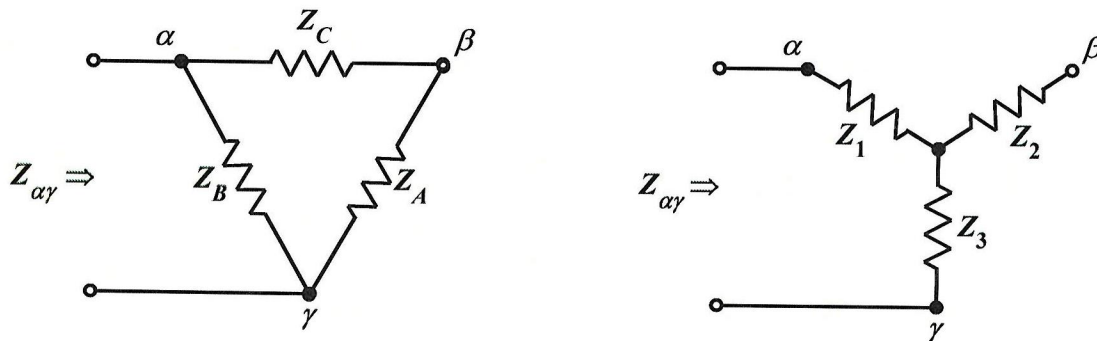
$$Z_A = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_B = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}$$

$$Z_C = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$

How Can We Derive the Transformation Equations?

Consider the two diagrams shown below. The equivalent impedance with respect to the $\alpha - \gamma$ terminal pair must be the same for both circuits if they are indeed equivalent.

$$Z_{\alpha\gamma} = Z_B \| (Z_A + Z_C) = \frac{Z_A Z_B + Z_B Z_C}{Z_A + Z_B + Z_C} \qquad \text{and} \qquad Z_{\alpha\gamma} = Z_1 + Z_3$$
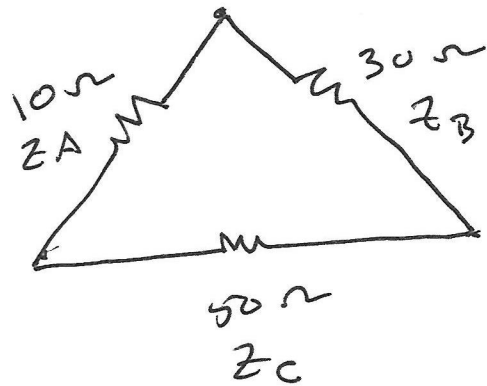
So,

$$Z_1 + Z_3 = \frac{Z_A Z_B + Z_B Z_C}{Z_A + Z_B + Z_C} \qquad (1)$$

Similarly,
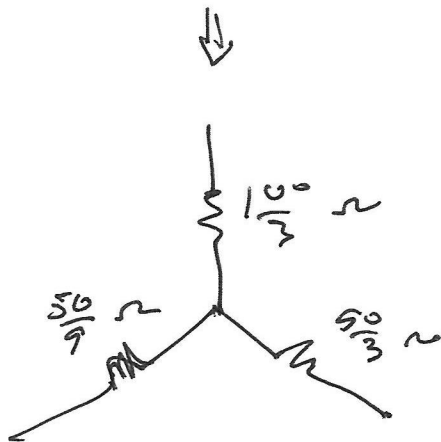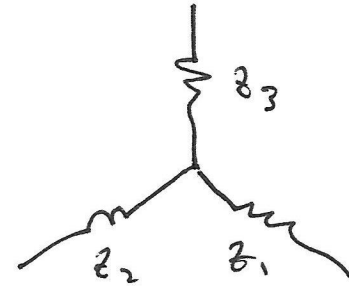
$$Z_1 = \frac{(30)(50)}{10 + 30 + 50} = \frac{1500}{90} = \frac{150}{9}$$

$$= \frac{50}{3} \ \Omega$$

$$Z_2 = \frac{(10)(50)}{90} = \frac{500}{90} = \frac{50}{9} \ \Omega$$

$$Z_3 = \frac{(10)(30)}{90} = \frac{300}{9} = \frac{100}{3} \ \Omega$$
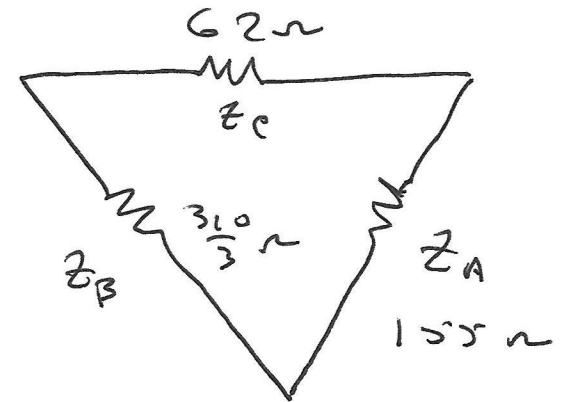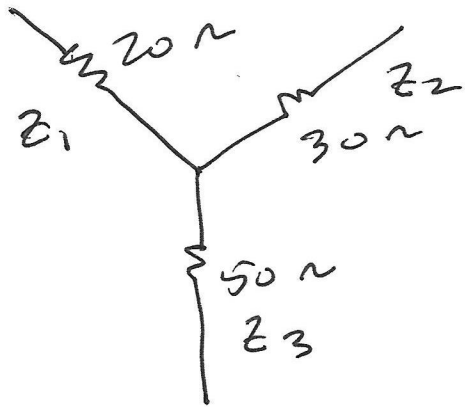
$$Z_A = \frac{(20)(30) + (30)(50) + (20)(50)}{20}$$

$$\approx \frac{600 + 1500 + 1000}{20}$$

$$= \frac{3100}{20} = 155\ \Omega$$

$$Z_B = \frac{3100}{30} = \frac{310}{3}\ \Omega$$

$$Z_C = \frac{3100}{50} = \frac{310}{5} = 62\ \Omega$$
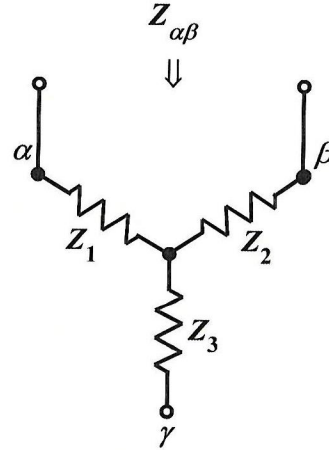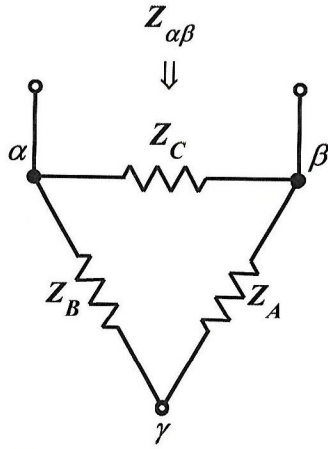
$$Z_{\alpha\beta} = Z_C \| (Z_A + Z_B) = \frac{Z_A Z_C + Z_B Z_C}{Z_A + Z_B + Z_C} \quad \text{and} \quad Z_{\alpha\beta} = Z_1 + Z_2$$

So,

$$Z_1 + Z_2 = \frac{Z_A Z_C + Z_B Z_C}{Z_A + Z_B + Z_C} \tag{2}$$

And, for the two diagrams shown below:
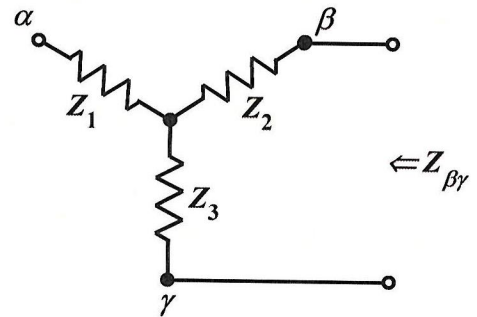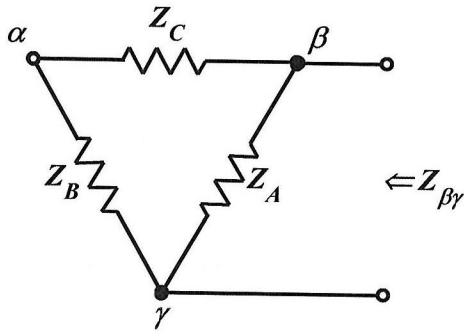
$$Z_{\beta\gamma} = Z_A \| (Z_B + Z_C) = \frac{Z_A Z_B + Z_A Z_C}{Z_A + Z_B + Z_C} \quad \text{and} \quad Z_{\beta\gamma} = Z_2 + Z_3$$

So,

$$Z_2 + Z_3 = \frac{Z_A Z_B + Z_A Z_C}{Z_A + Z_B + Z_C} \tag{3}$$

Then, $(1) + (2) - (3)$ gives

$$(Z_1 + Z_3) + (Z_1 + Z_2) - (Z_2 + Z_3) = \frac{(Z_A Z_B + Z_B Z_C) - (Z_A Z_C + Z_B Z_C) + (Z_A Z_B + Z_A Z_C)}{Z_A + Z_B + Z_C}$$

or $\qquad 2Z_1 = \dfrac{2Z_B Z_C}{Z_A + Z_B + Z_C} \qquad\qquad \Rightarrow \qquad\qquad Z_1 = \dfrac{Z_B Z_C}{Z_A + Z_B + Z_C}$

Similarly, $-(1)+(2)+(3)$ gives

$$-\left(Z_1+Z_3\right)+\left(Z_1+Z_2\right)+\left(Z_2+Z_3\right) = \dfrac{-(Z_A Z_B + Z_B Z_C)+(Z_A Z_C + Z_B Z_C)+(Z_A Z_B + Z_A Z_C)}{Z_A + Z_B + Z_C}$$

or $\qquad 2Z_2 = \dfrac{2Z_A Z_C}{Z_A + Z_B + Z_C} \qquad\qquad \Rightarrow \qquad\qquad Z_2 = \dfrac{Z_A Z_C}{Z_A + Z_B + Z_C}$

and $(1)-(2)+(3)$ yields

$$\left(Z_1+Z_3\right)-\left(Z_1+Z_2\right)+\left(Z_2+Z_3\right) = \dfrac{(Z_A Z_B + Z_B Z_C)-(Z_A Z_C + Z_B Z_C)+(Z_A Z_B + Z_A Z_C)}{Z_A + Z_B + Z_C}$$

or $\qquad 2Z_3 = \dfrac{2Z_A Z_B}{Z_A + Z_B + Z_C} \qquad\qquad \Rightarrow \qquad\qquad Z_3 = \dfrac{Z_A Z_B}{Z_A + Z_B + Z_C}$

This verifies the first set of transformation equations.

Now, consider the following: (The red lines designate a short circuit between the two terminals.)
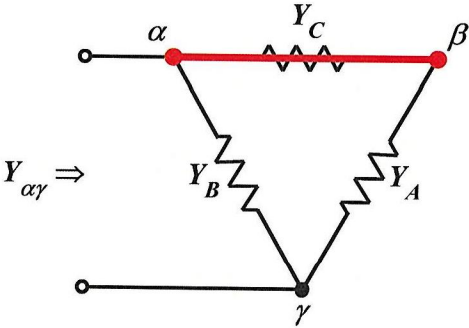
$$Y_{\alpha\gamma} = Y_A + Y_B \qquad \text{and} \qquad Y_{\alpha\gamma} = \frac{(Y_1 + Y_2)Y_3}{Y_1 + Y_2 + Y_3}$$
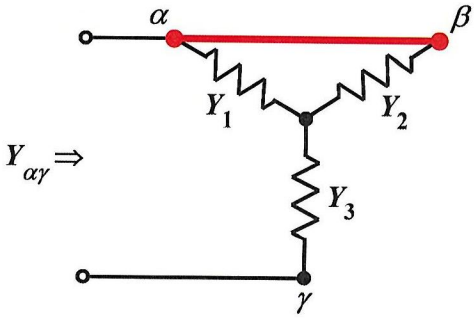
So,
$$Y_A + Y_B = \frac{Y_1Y_3 + Y_2Y_3}{Y_1 + Y_2 + Y_3} \qquad (4)$$

Similarly
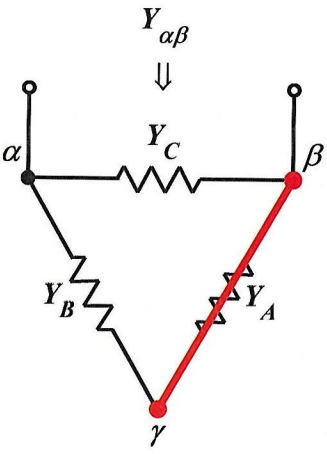
$$Y_{\alpha\beta} = Y_B + Y_C \qquad \text{and} \qquad Y_{\alpha\beta} = \frac{(Y_2 + Y_3)Y_1}{Y_1 + Y_2 + Y_3}$$
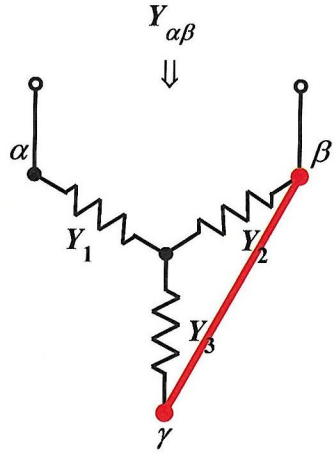
So,
$$Y_B + Y_C = \frac{Y_1Y_2 + Y_1Y_3}{Y_1 + Y_2 + Y_3} \qquad (5)$$

and
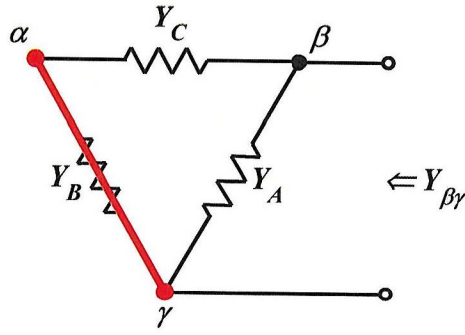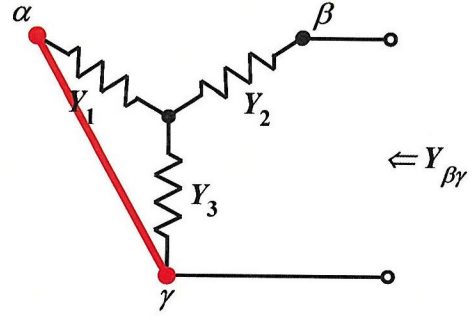
$$Y_{\beta\gamma} = Y_A + Y_C \qquad \text{and} \qquad Y_{\beta\gamma} = \frac{(Y_1 + Y_3)Y_2}{Y_1 + Y_2 + Y_3}$$

So,
$$Y_A + Y_C = \frac{Y_1 Y_2 + Y_2 Y_3}{Y_1 + Y_2 + Y_3} \qquad (6)$$

Now $(4) - (5) + (6)$ gives

$$(Y_A + Y_B) - (Y_B + Y_C) + (Y_A + Y_C) = \frac{(Y_1 Y_3 + Y_2 Y_3) - (Y_1 Y_2 + Y_1 Y_3) + (Y_1 Y_2 + Y_2 Y_3)}{Y_1 + Y_2 + Y_3}$$

or
$$2Y_A = \frac{2Y_2 Y_3}{Y_1 + Y_2 + Y_3} \qquad \Rightarrow \qquad Y_A = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}$$

Therefore,
$$Z_A = \frac{Y_1 + Y_2 + Y_3}{Y_2 Y_3} = \frac{Z_2 Z_3 + Z_1 Z_3 + Z_1 Z_2}{Z_1} \text{ or } Z_A = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$(4) + (5) - (6)$ gives

$$(Y_A + Y_B) + (Y_B + Y_C) - (Y_A + Y_C) = \frac{(Y_1 Y_3 + Y_2 Y_3) + (Y_1 Y_2 + Y_1 Y_3) - (Y_1 Y_2 + Y_2 Y_3)}{Y_1 + Y_2 + Y_3}$$

or
$$2Y_B = \frac{2Y_1 Y_3}{Y_1 + Y_2 + Y_3} \qquad \Rightarrow \qquad Y_B = \frac{Y_1 Y_3}{Y_1 + Y_2 + Y_3}$$

Therefore,
$$Z_B = \frac{Y_1 + Y_2 + Y_3}{Y_1 Y_3} = \frac{Z_2 Z_3 + Z_1 Z_3 + Z_1 Z_2}{Z_2} \text{ or } Z_B = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}$$

and $-(4) + (5) + (6)$ gives

$$-\left(Y_A + Y_B\right) + \left(Y_B + Y_C\right) + \left(Y_A + Y_C\right) = \frac{-\left(Y_1Y_3 + Y_2Y_3\right) + \left(Y_1Y_2 + Y_1Y_3\right) + \left(Y_1Y_2 + Y_2Y_3\right)}{Y_1 + Y_2 + Y_3}$$

or $\qquad 2Y_C = \dfrac{2Y_1Y_2}{Y_1 + Y_2 + Y_3} \qquad \Rightarrow \qquad Y_C = \dfrac{Y_1Y_2}{Y_1 + Y_2 + Y_3}$

Therefore, $\qquad Z_C = \dfrac{Y_1 + Y_2 + Y_3}{Y_1Y_2} = \dfrac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{Z_3}$ or $Z_C = Z_1 + Z_2 + \dfrac{Z_1Z_2}{Z_3}$

These results verify the second set of transformation equations.

Example 1:

Determine the equivalent resistance of the circuit shown.
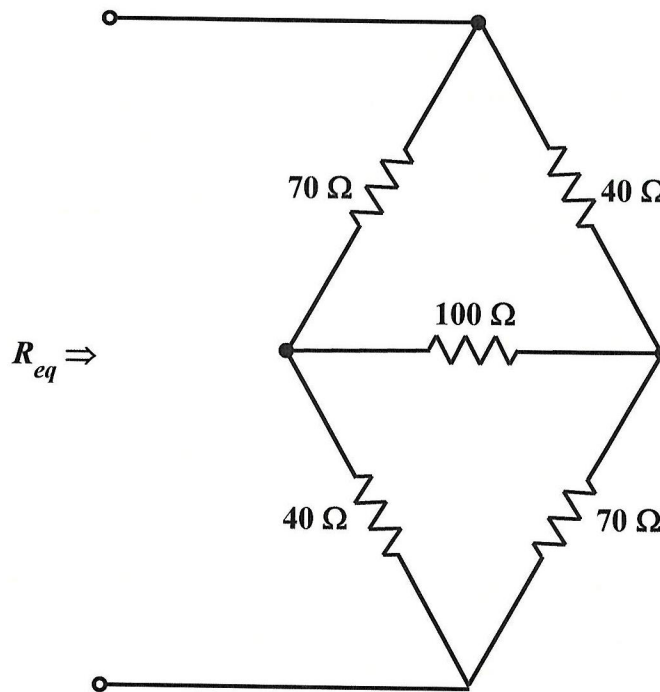
As is, determining the equivalent resistance is very difficult. However, by applying the $\Delta \rightarrow Y$ transformation equations discussed earlier, the problem can be greatly simplified. Looking at the bottom half of the circuit, and comparing with the notes above:

$Z_A = 600\ \Omega$, $Z_B = 300\ \Omega$, and $Z_C = 100\ \Omega$.

Applying the $\Delta \rightarrow Y$ transformation equations:

$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} = \frac{(300\ \Omega)(100\ \Omega)}{(600\ \Omega) + (300\ \Omega) + (100\ \Omega)} = \frac{30,000}{1000}\ \Omega = 30\ \Omega$$

$$Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} = \frac{(600\ \Omega)(100\ \Omega)}{(600\ \Omega) + (300\ \Omega) + (100\ \Omega)} = \frac{60,000}{1000}\ \Omega = 60\ \Omega$$

$$Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} = \frac{(600\ \Omega)(300\ \Omega)}{(600\ \Omega) + (300\ \Omega) + (100\ \Omega)} = \frac{180,000}{1000}\ \Omega = 180\ \Omega$$

Now, we have the following equivalent circuit:
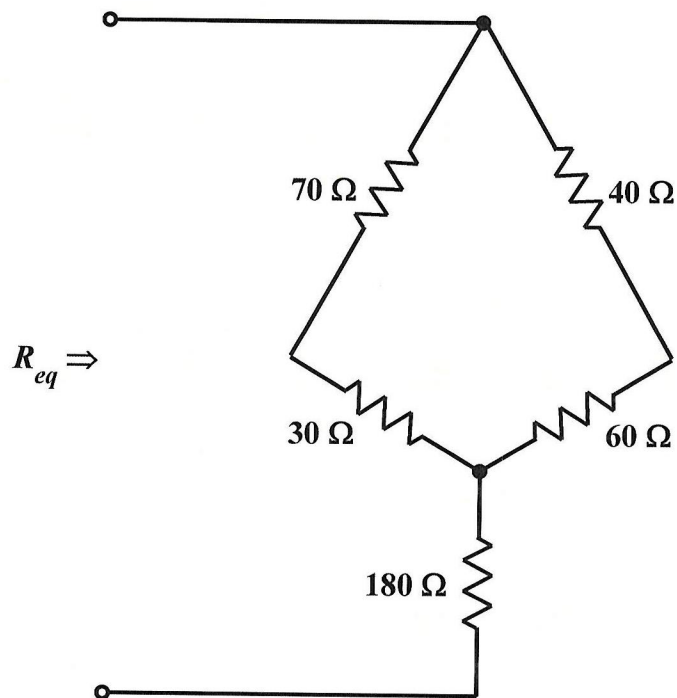
Here, determining the equivalent resistance is straightforward:

$$R_{eq} = \left\{ \left[(70\ \Omega) + (30\ \Omega) \right] \| \left[(40\ \Omega) + (60\ \Omega) \right] \right\} + (180\ \Omega)$$
$$= \left[(100\ \Omega) \| (100\ \Omega) \right] + (180\ \Omega)$$
$$= (50\ \Omega) + (180\ \Omega)$$
$$= 230\ \Omega$$

Example 2:

Determine the value of V_o in the circuit shown.
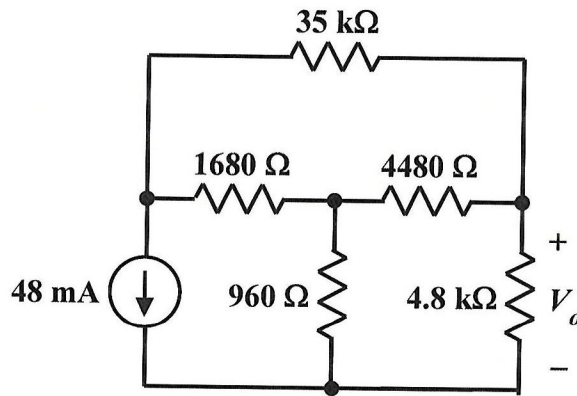
The three resistors in the middle of the circuit comprise a "wye" subcircuit, with $Z_1 = 1680\ \Omega$, $Z_2 = 4480\ \Omega$, and $Z_3 = 960\ \Omega$.

Applying the $\mathbf{Y} \rightarrow \mathbf{\Delta}$ transformation equations:

$$Z_A = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$= \frac{(1680\ \Omega)(4480\ \Omega) + (4480\ \Omega)(960\ \Omega) + (1680\ \Omega)(960\ \Omega)}{1680\ \Omega}$$

$$= \frac{13,440,000}{1680}\ \Omega = 8\ \text{k}\Omega$$

$$Z_B = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2} = \frac{13,440,000}{4480}\ \Omega = 3\ \text{k}\Omega$$

$$Z_C = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} = \frac{13,440,000}{960}\ \Omega = 14\ \text{k}\Omega$$

Now we have the following equivalent circuit:
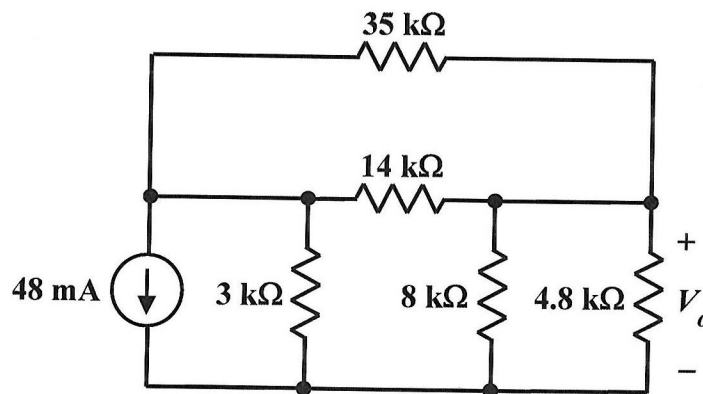
Now, note that the $35\ k\Omega$ and $14\ k\Omega$ resistors are in parallel and equivalent to $\dfrac{(35\ k\Omega)(14\ k\Omega)}{(35\ k\Omega)+(14\ k\Omega)}=10\ k\Omega$. Also, the $8\ k\Omega$ and $4.8\ k\Omega$ resistors are in parallel and equivalent to $\dfrac{(8\ k\Omega)(4.8\ k\Omega)}{(8\ k\Omega)+(4.8\ k\Omega)}=3\ k\Omega$. Thus, the circuit can be redrawn as follows;
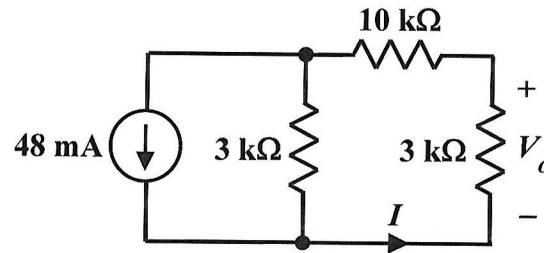
Hence, the circuit can be treated as a current divider with

$$I=\frac{3\ k\Omega}{(3\ k\Omega)+\left[(3\ k\Omega)+(10\ k\Omega)\right]}\cdot(48\ mA)=\frac{3}{16}\cdot(48\ mA)=9\ mA$$

and then

$$V_o=-(3\ k\Omega)\cdot I=-27\ V$$

Simulated Inductors

The Gyrator

In 1947 a Dutch engineer named Bernard Tellegen proposed a fifth linear, constant, passive network element (in addition to the capacitor, the resistor, the inductor, and the ideal transformer). He called it the gyrator, and described it as a linear two-port device that couples the current into one port to the voltage across the other and vice versa.
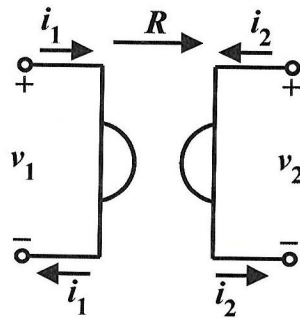
Figure 1.

His gyrator violates the reciprocity relation, and is represented by antisymmetric impedance and admittance matrices:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{gyrator} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & -R \\ R & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

where R is called the *gyration resistance*, and

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{gyrator} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where $Y_{gyrator} = Z_{gyrator}^{-1}$, and $G \triangleq \dfrac{1}{R}$ is called the *gyration conductance*. In other words, the instantaneous currents and voltages are related by

$$\begin{array}{ccc} v_2 = Ri_1 & & i_1 = Gv_2 \\ & \text{or} & \\ v_1 = -Ri_2 & & i_2 = -Gv_1 \end{array}.$$

The gyration resistance (or equivalently the gyration conductance) has an associated direction indicated by an arrow on the schematic diagram. By convention, the given gyration resistance or conductance relates the voltage across the port at the head of the arrow to the current at its tail. The voltage at the tail of the arrow is related to the current at its head by *minus* the stated resistance.

Reversing the arrow is equivalent to negating the gyration resistance, or to reversing the polarity of either port.

Although the gyrator is characterized by a "resistance" value, it is a lossless component. From the governing equations, the instantaneous power absorbed by a gyrator is identically zero as shown below.

$$P = v_1 i_1 + v_2 i_2 = \left(-R i_2\right) i_1 + \left(R i_1\right) i_2 = -R i_1 i_2 + R i_1 i_2 \equiv 0$$

Another important property of a gyrator is that it inverts the current–voltage characteristic of an electrical component or network. In the case of linear elements, the impedance is also inverted. In other words, a gyrator can make an inductor behave as a capacitor, as shown below:

Example 1
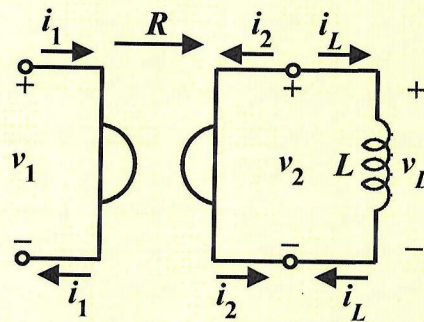
Figure 2.

$$v_L = L \frac{di_L}{dt}, \quad i_2 = -i_L, \quad v_2 = v_L$$

$$i_1 = G v_2 = G v_L = GL \frac{di_L}{dt} = GL \frac{d(-i_2)}{dt} = -GL \frac{d(-Gv_1)}{dt} = G^2 L \frac{dv_1}{dt}$$

which is the *i-v* relationship for a capacitor having value $C = G^2 L$.
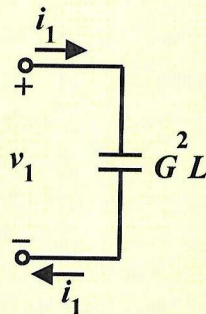
Figure 3.

Or, it can make a series LC circuit behave like a parallel LC circuit:

Example 2
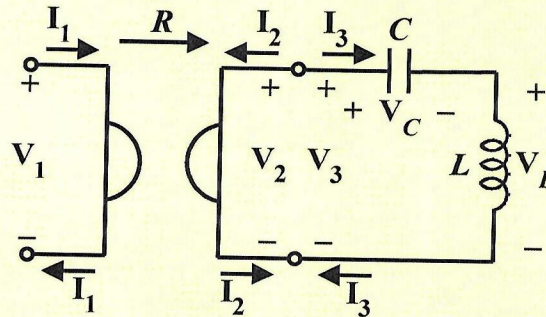
Figure 4.

Here:

$$V_2 = V_3, \quad V_3 = V_C + V_L, \quad V_C = \frac{1}{sC}I_3, \quad V_L = sLI_3, \quad I_3 = I_2$$

so that:

$$I_1 = GV_2 = GV_3 = G(V_L + V_C) = G\left(sLI_3 + \frac{1}{sC}I_3\right)$$

$$= G\left(sL + \frac{1}{sC}\right)I_3 = -G\left(sL + \frac{1}{sC}\right)I_2 = G^2\left(sL + \frac{1}{sC}\right)V_1$$

which describes a capacitance, $C = G^2L$, in parallel with an inductor, $L = \dfrac{C}{G^2} = R^2C$, with voltage V_1 across the pair.
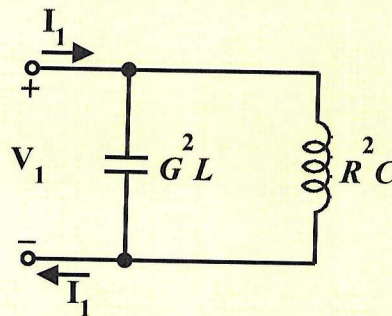
Figure 5.

and so on.

Relationship to the Ideal Transformer

An ideal gyrator is similar to an ideal transformer in that it is a two-port linear, lossless, passive, memoryless device. However, the transformer couples the voltage across port 1 to the voltage across port 2, and the current into port 1 to the current into port 2, while the gyrator cross-couples voltage to current and current to voltage.

Example 3

Cascading two gyrators with gyration resistances R_1 and R_2 achieves a voltage-to-voltage coupling equivalent to that of an ideal transformer having turns ratio $R_1 : R_2$.

$$v_2 = R_2 i_{21} = -R_2 i_{12} = R_2 G_1 v_1 = \frac{R_2}{R_1} v_1$$

(a)

$$v_2 = \frac{n_2}{n_1} v_1$$

(b)
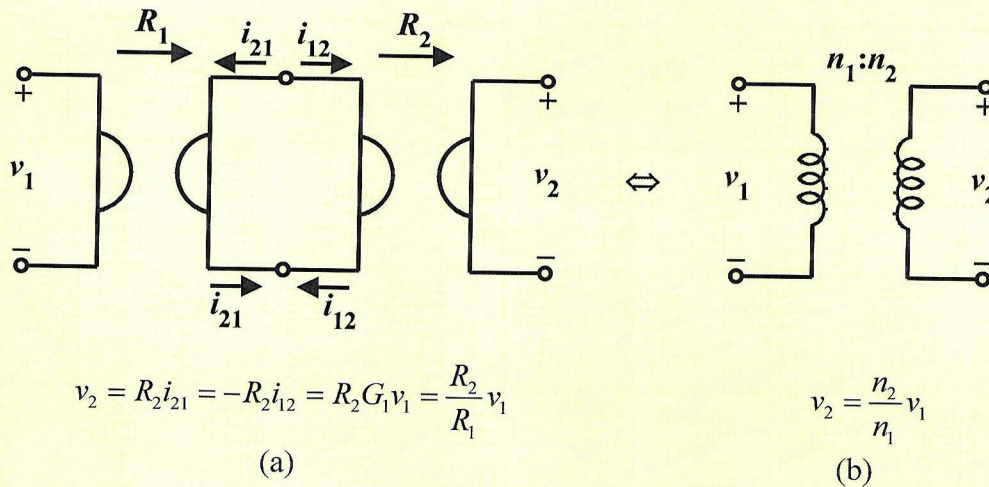
Figure 6.

Hence, from the point of view of network theory, transformers are redundant when gyrators are available.

Why do we Need this Thing?

Inductors do not lend themselves easily to the miniaturization and monolithic fabrication requirements that dominate today's electronics industry. They're naturally bulky, and being made from a coil of wire, tend to generate electromagnetic interference, and are susceptible to that generated elsewhere (e.g., power-line hum). Fortunately, Tellegen's gyrator allows us a way to overcome this problem

Example 4
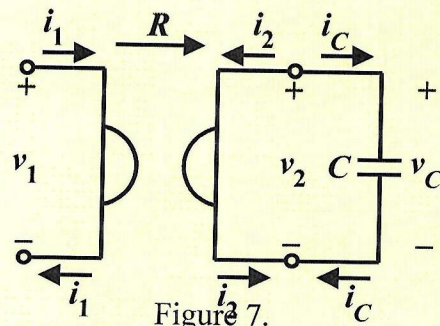
Figure 7.

Note that:

$$i_C = C\frac{v_C}{dt}, \qquad i_2 = -i_C, \qquad v_2 = v_C$$

And, then:

$$v_1 = -Ri_2 = -Ri_C = -RC\frac{d(-v_C)}{dt} = RC\frac{dv_2}{dt} = RC\frac{d(Ri_1)}{dt} = R^2C\frac{di_1}{dt}$$

which is the *v-i* relationship for an inductor having value $L = R^2C$.
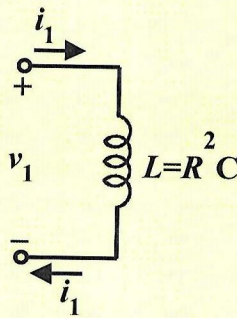
Figure 8.

Thus, that bulky, heavy, expensive coil of wire (the inductor) can be replaced by a much smaller assembly containing a capacitor and a gyrator. That is, anything that can be built from resistors, capacitors, inductors, transformers and gyrators, can also be built using just resistors, gyrators and inductors (or capacitors).

So Where do We Get a Gyrator?

According to Tellegen, "The gyrator can be realized by means of a medium consisting of particles carrying both permanent electric and permanent magnetic dipoles or by means of a gyromagnetic effect of a ferromagnetic medium."

Fortunately, at low frequencies and low power, the behavior of the gyrator can alternately be accomplished with a small op-amp circuit. Hence, an inductor can be replaced by a much smaller assembly containing a capacitor, operational amplifiers, and resistors.

Thus, for example, an RLC bandpass filter can be realized with capacitors, resistors and operational amplifiers without using inductors.

Simulated Inductor

An example of a gyrator simulating inductance, with an approximate equivalent circuit, is shown below. The two Z_{in} have similar values in typical applications.
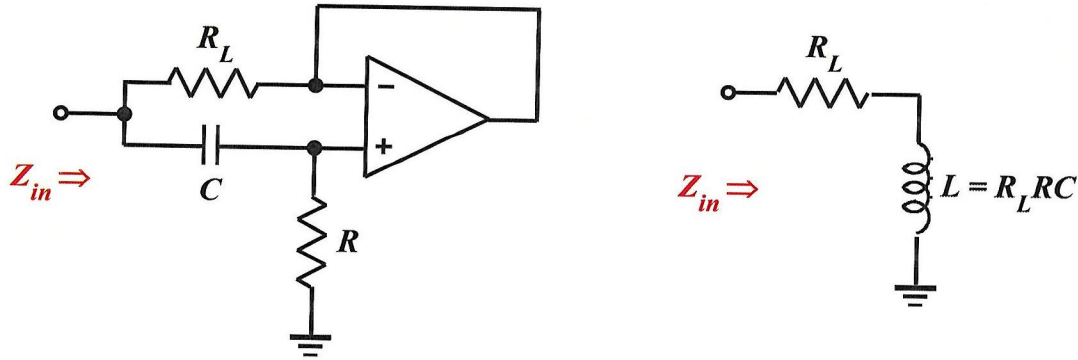
Figure 9. A Floating Simulated Inductor.

Operation

In the circuit shown, one port of the gyrator is between the input terminal and ground, while the other port is terminated with the capacitor. The circuit works by inverting and multiplying the effect of the capacitor in an RC differentiating circuit where the voltage across the resistor R behaves through time in the same manner as the voltage across an inductor. The op-amp follower buffers this voltage and applies it back to the input through the resistor R_L. The desired effect is an impedance of the form of an ideal inductor L with a series resistance R_L:

$$Z = R_L + j\omega L$$

From the diagram, the input impedance of the op-amp circuit is:

$$Z_{in} = \left(R_L + j\omega R_L RC \right) \middle\| \left(R + \frac{1}{j\omega C} \right)$$

With $R_L RC = L$, it can be seen that the impedance of the simulated inductor is the desired impedance in parallel with the impedance of the RC circuit. In typical designs, R is chosen to be sufficiently large such that the first term dominates; thus, the RC circuit's effect on input impedance is negligible.

$$Z_{in} \approx R_L + j\omega R_L RC$$

This is the same as a resistance R_L in series with an inductance $L = R_L RC$. There is a practical limit on the minimum value that R_L can take, determined by the current output capability of the op-amp.

The impedance cannot increase indefinitely with frequency, and eventually the second term limits the impedance to the value of R.

(a) "Real" Inductor

(b) "Simulated" Inductor
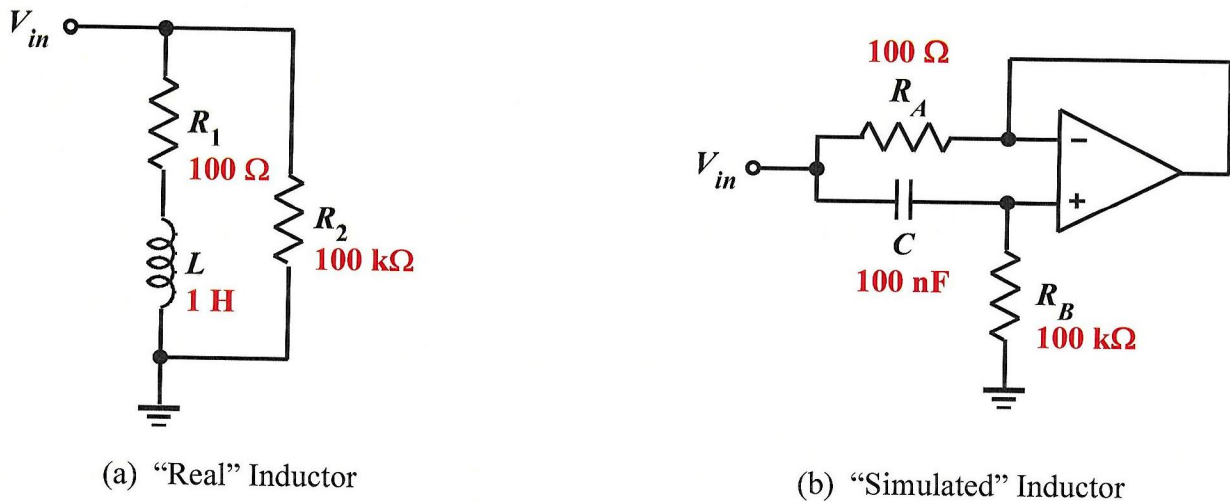
Figure 10. Equivalent Circuits(?)

The circuit is much smaller than a real inductor (at least for the larger values), and does not suffer from noise pickup. It does have a limited Q (quality factor), but it is rare that very high Q circuits are needed in audio, so this is not really a problem. It is also variable over a moderately wide range, something that is very difficult with the wire wound genuine article. R_2 in parallel with the inductor 'circuit' is rarely shown in 'equivalence' diagrams, but if you want an accurate representation it must be included. The simulated and real inductors perform identically once the parallel resistance is included.

Parallel resistor R_2 is frequently omitted from the real inductor model because its value is so large. However, performance of the simulated and real inductor models is more easily matched if R_2 is included.

The inductance, in Henrys, for the simulator is

$$L = R_A(R_B - R_A)C$$

Since $R_A \ll R_B$, this can be approximated by

$$L = R_A R_B C$$

with very little error. In fact, a ratio of 100:1 would normally be the *lowest* one would encounter in practice, and this simplification would yield an error of only 1% at most. Component tolerances would have a much more significant effect.

Example 5

For the circuit shown in Figure 10(b), the simulated inductance is

$$L = R_A (R_B - R_A) C = 100 \times \left[\left(100 \times 10^3 \right) - 100 \right] \times \left(100 \times 10^{-9} \right) = 0.999 \text{ H},$$

and the approximation is

$$L = R_A R_B C = 100 \times \left(100 \times 10^3 \right) \times \left(100 \times 10^{-9} \right) = 1 \text{ H}.$$

The resulting error is only

$$\text{Error} = \frac{1 - 0.999}{0.999} \times 100 \approx 0.1 \%.$$

The value of the inductor in Example 1 is large, and it would be very expensive and bulky if made conventionally. It would require a large iron core to be able to withstand even a small amount of DC, and the inductance would change depending on how much DC is present.

It should be noted that the simulated inductor also has problems because its current is limited by the current capability of the opamp, so it can handle only up to ±20 mA or so with no change in performance.

Even so, the simulated inductor is an important building block for such things as equalizers and spectrum displays, and is quite common in audio applications.

How Does it Work?

The idea is very simple, but operation is less easy to understand. Essentially the circuit uses a capacitor, and 'reverses' its operation, thus making an 'inductor'. The essential character of an inductor is that it resists any change in its current, so if a DC voltage is applied to an inductance, the current will rise slowly, and the voltage will fall until the internal resistance becomes significant.

An inductor also passes low frequencies more readily than high frequencies - the opposite of a capacitor. An ideal (that word again) inductor has zero resistance, so will pass DC with no limitation, but will have an infinitely high impedance at infinite frequency. These limits are generally considered to be outside the audio range.

To understand how the circuit works, remember that the output of the opamp is (almost) exactly the same as the non-inverting input. Imagine a DC voltage of 1 V is suddenly applied to the input,

via resistor R_1. The opamp will ignore the sudden load because the change is coupled directly to the input via C. The opamp will represent a high impedance. Just as an inductor would do.

With the passage of time, C charges via R_2, the voltage across R_2 falls, the opamp sees less and less of the input signal, and starts to draw current from the input via R_1. This continues as the capacitor approaches full charge, and the opamp has close to zero input, so the output is also close to zero volts.

Eventually resistor R_1 becomes the only limiting factor to current flow, and this appears as a series resistance within the inductor in the same way as the resistance of the wire in a real inductor behaves. This series resistance limits the available Q of both the simulated and real inductor, with the main difference being the magnitude - real inductors generally have less resistance than the simulated variety, but with the high inductance values often needed for audio this may not be true. Inductance is measured in Henrys, and for the simulated inductor is approximately equal to

$$L = R_1 R_2 C$$

A more accurate version of the formula (due to Siegfried Linkwitz) is shown below, but normally the error from the simple version will be very low with typical values - a ratio of 100:1 would normally be the lowest one would use, and this will have an error of only 1%. Component tolerance will have more effect, but for completeness, here is the accurate version ...

$$L = R_1 \left(R_2 - R_1 \right) C$$

... so for the circuit shown is 1 Henry. This is a large inductance, and would be very expensive and bulky if made conventionally. The real inductance will have lower resistance and higher Q, but will need to have a large iron core to be able to withstand even a small amount of DC, and the inductance will change depending on how much DC is present. The simulated inductor is limited by the current capability of the opamp, so can handle up to ±20mA with no change in performance. There are some limitations to the simulated inductor:

- First (and most annoying) is that one end of the inductor is earthed. Although simulated inductors have been made that are floating (can be connected in any way you like), these are expensive and uncommon. Fortunately the standard version is quite suitable for many audio applications, so this is not too great a burden.
- The simulated inductor cannot be made with high Q, since the value of R_1 cannot be made low enough to allow a Q of more than about 10. This is due to the limitations of the opamp - a minimum value of 100 Ω is usually specified for R_1, although lower values are sometimes used. This represents a series resistance (equivalent to wire resistance in a real inductor).
- There is also a resistance effectively in parallel with the simulated inductor, equal to the value of $R_1 + R_2$. Although this can be measured, it is not generally a hindrance to practical circuit design.

- Although the simulated inductor acts in many ways like the real thing, it does not have the same energy storage, and cannot respond like a proper wound inductor. When the input voltage is suddenly removed from a real inductor, the collapse of the magnetic field causes a large voltage pulse of the opposite polarity - this does not happen properly with a simulated inductor, since there is no magnetic field involved. The simulated inductor will still try, but the back-EMF is limited to the voltage swing of the opamp, so the flyback pulse is limited to this value.

Example 6

Figure 11 shows two simple LC filters. The left circuit uses a real inductor, and the right circuit has a simulated inductor. Resistor $R_{1,1}$ is the coil winding resistance – equivalent to $R_{1,2}$ in the gyrator circuit. A series resonant circuit has minimum impedance at resonance, and in the configuration shown will act as a notch filter, reducing the transmitted signal level at the resonant frequency.

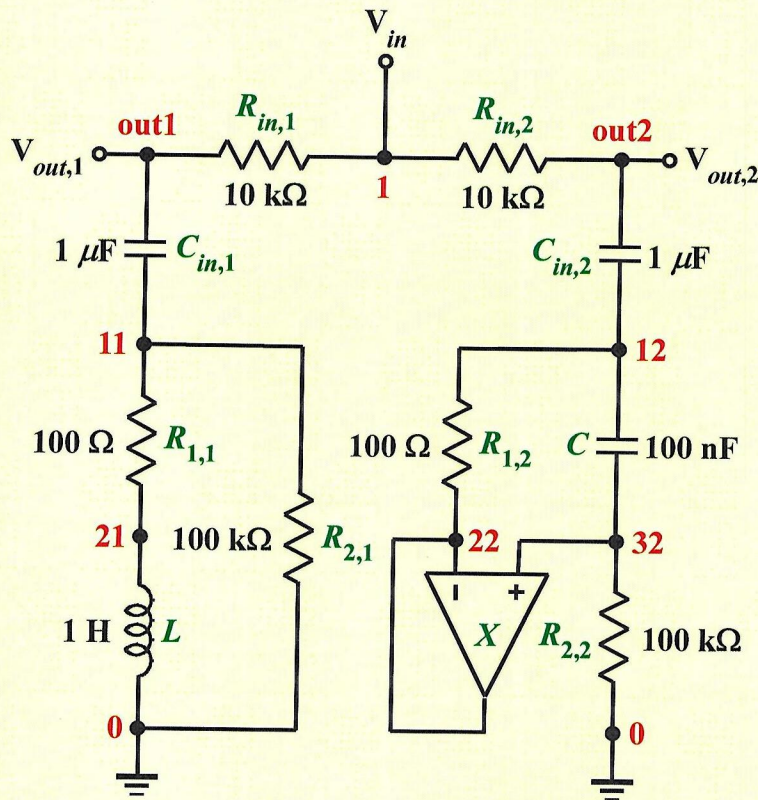
Figure 11. Two LC Bandstop Filters, with Real (left) and Simulated (right) Inductors.

The notch frequency for these filters is:

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times\left(1\times10^{-6}\right)}} \approx 159 \text{ Hz}.$$

A SPICE netlist for simulation of the two circuits is shown below. The resulting plot in Figure 12 shows the frequency response magnitude and phase angle for both circuits over the range $1\,\text{Hz} \le f \le 100\,\text{kHz}$. The reference level (near the top of the graph) is 0 dB, and neither circuit exhibits any appreciable loss outside the stopband. In fact, the two are so close together that distinguishing one from the other is nearly impossible.

```
* Q:\Websites\RES\EE 3340\Simulated Inductor.cir

Vin 1   0   AC  1   0

* "Real" Inductor
Rin1     1    out1     10k
Cin1     out1     11   1u
R11 11   21   100
L    21  0    1
R21 11   0    100k

* Simulated Inductor
.subckt opamp     1    2    3
E    3   0    1    2    1T
.ends    opamp
Rin2     1    out2     10k
Cin2     out2     12   1u
R12 12   22   100
C    12  32   100n
X    32  22   22   opamp
R22 32   0    100k

.AC DEC 10   1    100k
.end
```
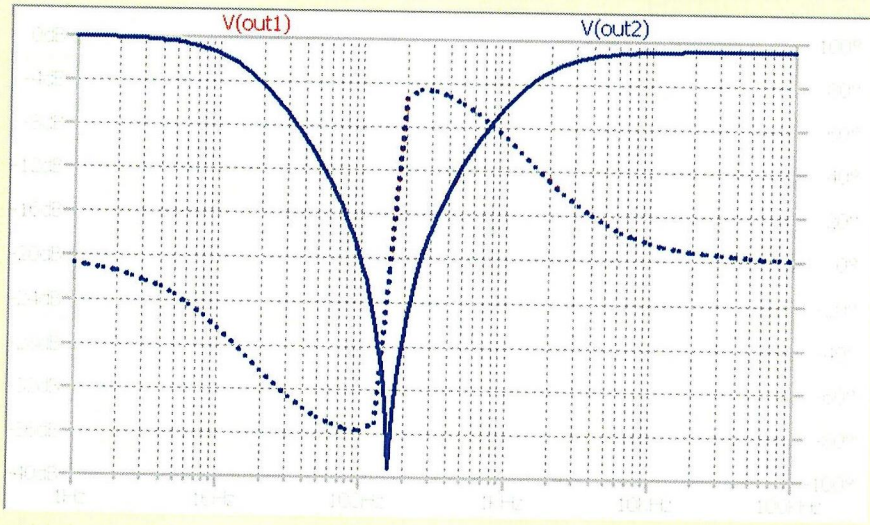

159 Hz

Figure 12. Frequency Response of the Circuits Shown in Figure 11.

Comparison with Actual Inductors

Simulated elements are electronic circuits that imitate actual elements. Simulated elements cannot replace physical inductors in all the possible applications as they do not possess all the unique properties of physical inductors.

There are some significant limitations to the simulated inductors discussed so far:

- First (and most annoying) is that one end of the inductor is grounded. Simulated floating inductors can be made, as we will see shortly, but they are more expensive. Fortunately, the grounded version is quite suitable for many applications, so this is often not a problem.

- The current in typical electric circuits is confined to the circuit, with very little "leakage". In typical magnetic circuits the magnetic field is not confined to the magnetic circuit because magnetic permeability also exists in any outside material. Thus, there may be significant "leakage flux" in the space outside the magnetic core. If the leakage flux is small compared to the main circuit, it can often be represented as additional elements. However, in extreme cases, a lumped-element model may not be appropriate at all, and field theory must be used instead.

- Magnetic circuits are nonlinear; the reluctance in a magnetic circuit is not constant, as resistance is, but varies depending on the magnetic field. At high magnetic fluxes the ferromagnetic materials used for the cores of magnetic circuits saturate, limiting further increase of the magnetic flux, so above this level the reluctance increases rapidly. In addition, ferromagnetic materials suffer from hysteresis so the flux in them depends not just on the instantaneous MMF but also on the history of that MMF. After the source of the magnetic flux is removed (or turned off), residual magnetism is left in ferromagnetic materials, creating flux with no MMF.

- Although the simulated inductor acts in many ways like the real thing, it does not have the same energy storage capability, and cannot respond like a wound inductor. When the input voltage is suddenly removed from a real inductor, the collapse of the magnetic field causes a large voltage pulse of the opposite polarity. This does not happen with a simulated inductor, since there is no magnetic field involved. The simulated inductor will still try, but the pulse magnitude is limited to the voltage swing of the opamp.

Magnitudes. In typical applications, both the inductance and the resistance of the gyrator are much greater than that of a physical inductor. Gyrators can be used to create inductors from the microhenry range up to the megahenry range. Physical inductors are typically limited to tens of henries, and have parasitic series resistances from hundreds of microhms through the low kilohm range. The parasitic resistance of a gyrator depends on the topology, but with the topology shown, series resistances will typically range from tens of ohms through hundreds of kilohms.

Quality. Physical capacitors are often much closer to "ideal capacitors" than physical inductors are to "ideal inductors". Because of this, a synthesized inductor realized with a gyrator and a capacitor may, for certain applications, be closer to an "ideal inductor" than any (practical) physical inductor can be. Thus, use of capacitors and gyrators may improve the quality of filter networks that would otherwise be built using inductors. Also, the Q factor of a synthesized inductor can be

selected with ease. The Q of an LC filter can be either lower or higher than that of an actual LC filter – for the same frequency, the inductance is much higher, the capacitance much lower, but the resistance also higher. Gyrator inductors typically have higher accuracy than physical inductors, due to the lower cost of precision capacitors than inductors.

Energy Storage. Simulated inductors do not have the inherent energy storing properties of the real inductors and this limits the possible power applications. The circuit cannot respond like a real inductor to sudden input changes (it does not produce a high-voltage back EMF); its voltage response is limited by the power supply. Since gyrators use active circuits, they only function as a gyrator within the power supply range of the active element. Hence gyrators are usually not very useful for situations requiring simulation of the 'flyback' property of inductors, where a large voltage spike is caused when current is interrupted. A gyrator's transient response is limited by the bandwidth of the active device in the circuit and by the power supply.

Externalities. Simulated inductors do not react to external magnetic fields and permeable materials the same way that real inductors do. They also don't create magnetic fields (and induce currents in external conductors) the same way that real inductors do. This limits their use in applications such as sensors, detectors and transducers.

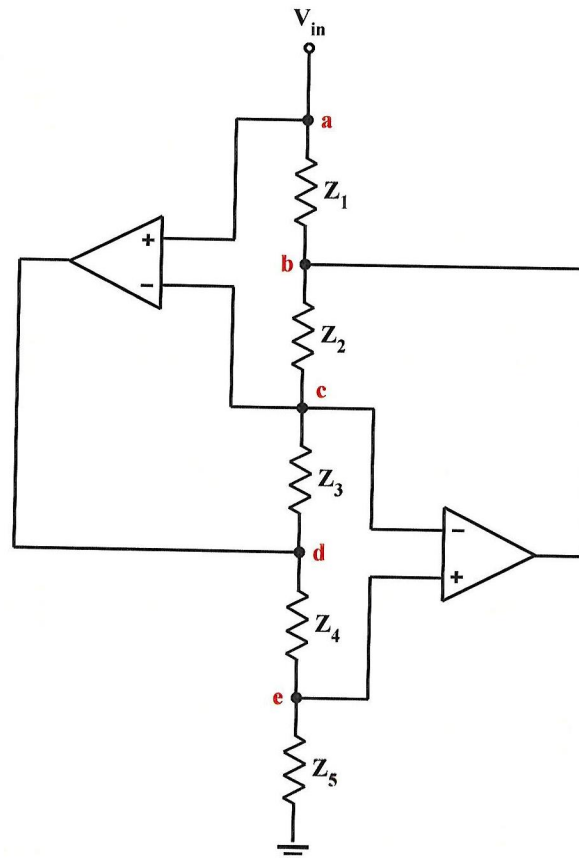Another approach, sometimes called an ***impedance converter*** is shown in Figure 13.

Figure 13. Impedance Converter with Equivalent Impedance: $Z_{in} = \dfrac{Z_1 Z_3 Z_5}{Z_2 Z_4}$.

If impedances Z_1, Z_2, Z_3 and Z_5 are resistors, and Z_4 is a capacitor, then

$$Z_{in} = \frac{R_1 R_3 R_5}{R_2 \left(\dfrac{1}{sC}\right)} = s\left(\frac{R_1 R_3 R_5 C}{R_2}\right)$$

which corresponds to a simulated inductance, $L = \dfrac{R_1 R_3 R_5 C}{R_2}$ or, if all resistors are the same, $L = R^2 C$.

> **Example 7**
>
> For example, if $R_1 = R_2 = R_3 = R_5 = 10\ \text{k}\Omega$ and $C = 0.01\ \mu\text{F}$ in the circuit of Figure 13, then
>
> $$L = \frac{\left(10 \times 10^3\right)^3 \left(10^{-2} \times 10^{-6}\right)}{10^4} = 1\ \text{H}$$

Grounding. The fact that one side of the simulated inductor is grounded restricts the possible applications (real inductors are floating). This limitation may preclude its use in some low-pass and notch filters. However, the gyrator can be used in a floating configuration with another gyrator so long as the floating "grounds" are tied together. This allows for a floating gyrator, but the inductance simulated across the input terminals of the gyrator pair must be cut in half for each gyrator to ensure that the desired inductance is met (the impedance of inductors in series adds together). This is not typically done as it requires even more components than in a standard configuration and the resulting inductance is a result of two simulated inductors, each with half of the desired inductance.

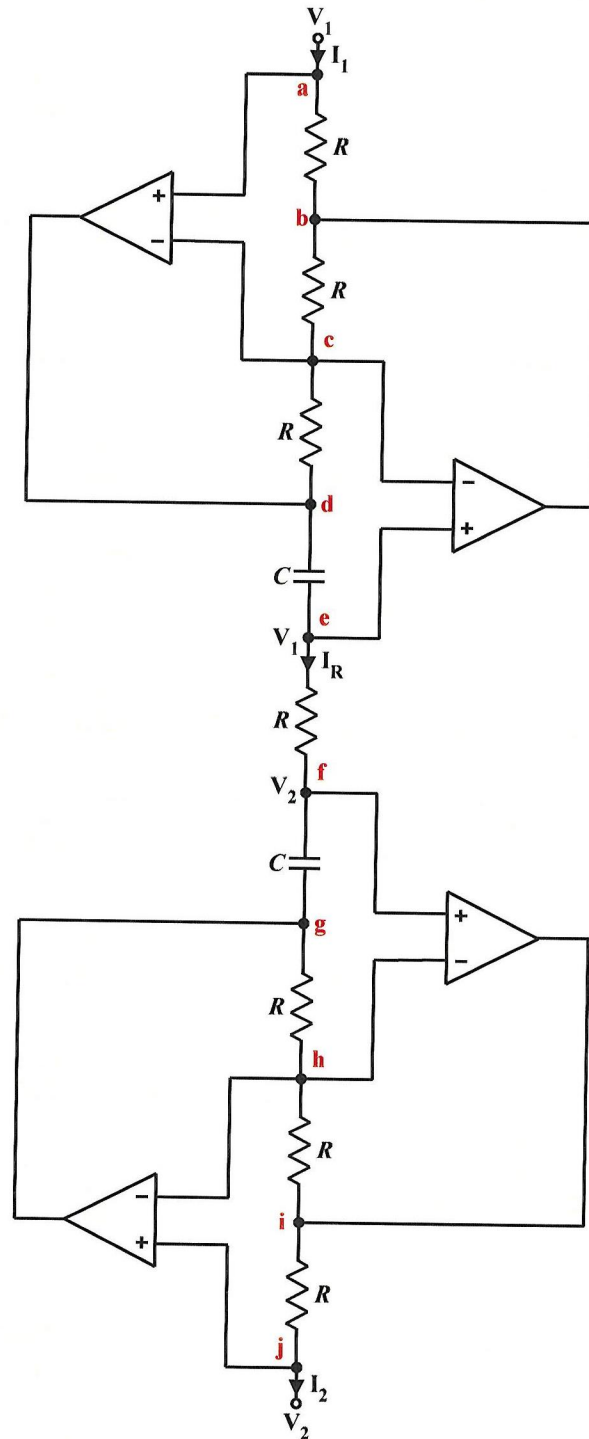A *floating* simulated inductor can be made as follows:

Figure 14. A Floating Simulated Inductor.

Here $Z = \dfrac{V_1 - V_2}{I_1} = sCR^2$, which again corresponds to a simulated inductance, $L = CR^2$.

* SCAM   Symbolic Circuit Analysis with MATLAB

     Need   Symbolic Toolbox

            Control System Toolbox

JNAP